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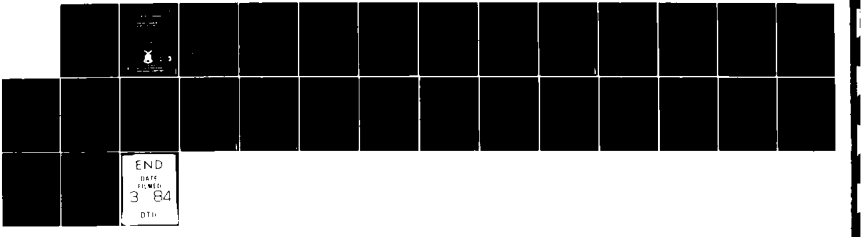

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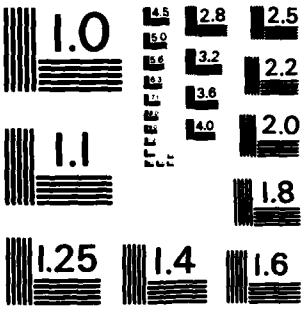

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DYNA-METRIC EXTENSIONS:

PARTIALLY MISSION CAPABLE DUE TO SUPPLY

ROBERT C. RUE, MAJOR, USAF

AUGUST 1982

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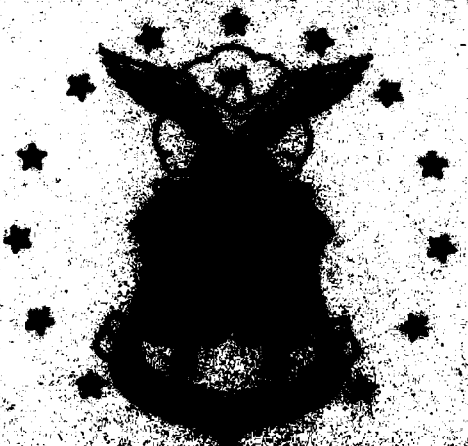

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Dyna-METRIC models repairable aircraft parts and assumes that the failure of any part would cause the aircraft to be Not Mission Capable due to Supply (NMCS). That assumption is relaxed in this paper to allow parts to be classified as essential, those whose failure would cause the aircraft to be NMCS, and nonessential, those whose failure would cause an aircraft to be Partially Mission Capable due to Supply (PMCS). Formulas are developed for

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computing the expected number of aircraft NMCS, PMCS, or both at any time for each of two maintenance policies. The first policy does not allow good parts to be removed from an aircraft, whereas the second does. Also, the results for two sets of parts are extended to allow the nonessential class to be further divided.

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DYNA-METRIC EXTENSIONS:
PARTIALLY MISSION CAPABLE
DUE TO SUPPLY

by

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PREFACE

This research was done for the Air Force Logistics Management Center at Gunter Air Force Station, Alabama and is part of an ongoing effort by several officers in the Air Force Academy's Department of Mathematical Sciences.

SUMMARY

Since the Air Force spends large amounts of money on aircraft spare parts, it is interested in ensuring the money is spent properly. To help determine how to invest in inventory the Air Force uses models of inventory systems. One of these models currently being evaluated by the Air Force Logistics Management Center is called Dyna-METRIC.

Dyna-METRIC models repairable aircraft parts and assumes the failure of any part would cause the aircraft to be Not Mission Capable due to Supply (NMCS). That assumption is relaxed in this paper to allow parts to be classified as essential, those whose failure would cause the aircraft to be NMCS, and nonessential, those whose failure would cause an aircraft to be Partially Mission Capable due to Supply (PMCS). I developed formulas for computing the expected number of aircraft NMCS, PMCS, or both at any time for each of two maintenance policies. The first policy does not allow good parts to be removed from an aircraft (no cannibalization), whereas the second does (full cannibalization).

In the last part of the paper, the results for the two sets are extended to allow the nonessential class of parts to be further divided.

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SECTION 1

INTRODUCTION AND BACKGROUND

The Logistics Management Center (LMC) is interested in studying the requirement for aircraft spare parts in a dynamic environment such as the start of a military deployment. One of the models the LMC is using for this study is Dyna-METRIC, a computer model developed by the Rand Corporation. Dyna-METRIC models the inventory and repair system for repairable aircraft parts. Dyna-METRIC developed from other models such as METRIC [3] and MOD-METRIC [2] and has some of its mathematical foundations presented in a Rand Note [1]. This paper presents the results of work done on relaxing one of the assumptions in the Rand Note, namely, the assumption that the failure of any aircraft part causes the aircraft to become unusable or Not Mission Capable due to Supply (NMCS) until the part is replaced. How the assumption is relaxed will be explained shortly.

An inventory and repair system supports a number of aircraft, NA(t), which is a function of time since aircraft may be added to the system or lost through attrition or reassignment. An aircraft is composed of N items, each of which is repairable. These parts are subject to failure, assumed to be independent of other failures, causing a demand for spares. The inventory system has $S_i(t)$ spares for part i, where i = 1, . . . , N. The number of spares is a function of time because aircraft use spares and the repair system produces spares. The Rand Note develops expressions that can be generalized to help develop new results. In this paper the classification of the N parts on an aircraft is expanded to consider parts

as essential, those whose failure would cause the aircraft to be NMCS; or nonessential, those whose failure would limit use of the aircraft but not ground it, causing the aircraft to be Partially Mission Capable due to Supply (PMCS).

This study further considers two classification systems for aircraft parts. In the first, parts are assigned to two mutually exclusive sets, those whose failure would cause an aircraft to be NMCS and those whose failure would cause an aircraft to be PMCS. In the second classification system, the PMCS category of parts is further divided.

The study also considers two of the maintenance policies presented in the Rand Note [1]: full cannibalization and no cannibalization. Under the full cannibalization policy, an aircraft that is NMCS or PMCS will be used as a source of spare parts for other aircraft. Under the no cannibalization policy, spare parts can only come from the stockpile of spares or from the repair shop.

Since the results presented in this paper are extensions of results in the Rand Note, the development here closely follows the Rand Note. The results for the first classification system, only two sets of parts, are presented in Section 2. Results for both maintenance policies will be presented before moving on to the second classification system in Section 3.

SECTION 2

TWO SETS OF PARTS

The N recoverable items that compose an aircraft are split into two mutually exclusive sets: I_1, the set of essential aircraft parts whose failure would cause an aircraft to be NMCS; and I_2, the set of nonessential items whose failure would cause an aircraft to be PMCS. Denote by NA(t) the number of aircraft supported by the inventory and repair system at time t. Let EN(t) be the average number of aircraft NMCS, EP(t) be the average number of aircraft PMCS, and ENP(t) be the average number of aircraft both NMCS and PMCS, all at time t. Expressions for EN(t), EP(t), and ENP(t) will be developed for the no cannibalization policy first.

No Cannibalization

If k shortages of part i exist, then the probability that an arbitrary aircraft has a shortage of part i is $\frac{k}{NA(t)}$. Thus the probability that an arbitrary aircraft has a shortage of part i at time t is $\frac{EB_i(t)}{NA(t)}$, where $EB_i(t)$ is the expected number of back orders of item i at time t as given in the Rand Note [1]. Since the failure of any part in set I_1 will cause an aircraft to be NMCS, the probability of an aircraft being NMCS is

$$\text{Prob}(\text{NMCS}(t)) = 1 - \prod_{i \in I_1} \left(1 - \frac{EB_i(t)}{NA(t)}\right).$$

Similarly, the probability of an aircraft being PMCS is

$$\text{Prob}(\text{PMCS}(t)) = 1 - \prod_{i \in I_2} \left(1 - \frac{EB_i(t)}{NA(t)}\right).$$

Since the failures are independent, the probability that an aircraft is both NMCS and PMCS is

$$\begin{aligned}\text{Prob}(\text{NMCS}(t) \cap \text{PMCS}(t)) &= \text{Prob}(\text{PMCS}(t)) \cdot \text{Prob}(\text{NMCS}(t)) \\ &= \{1 - \prod_{i \in I_1} (1 - \frac{EB_i(t)}{NA(t)})\}\{1 - \prod_{i \in I_2} (1 - \frac{EB_i(t)}{NA(t)})\} \qquad (1) \\ &= 1 - \prod_{i \in I_1} (1 - \frac{EB_i(t)}{NA(t)}) - \prod_{i \in I_2} (1 - \frac{EB_i(t)}{NA(t)}) + \prod_{i=1}^{N} (1 - \frac{EB_i(t)}{NA(t)}) \\ &= \text{Prob}(\text{NMCS}(t)) + \text{Prob}(\text{PMCS}(t)) - \text{Prob}(\text{NMCS}(t) \cup \text{PMCS}(t)),\end{aligned}$$

where $\text{Prob}(\text{NMCS}(t) \cup \text{PMCS}(t)) = 1 - \prod_{i=1}^{N} (1 - \frac{EB_i(t)}{NA(t)})$.

These probabilities together with the number of aircraft supported at time t, NA(t), can be used to compute the desired averages, EN(t), EP(t), and ENP(t). Thus

$$EN(t) = NA(t)[1 - \prod_{i \in I_1} (1 - \frac{EB_i(t)}{NA(t)})], \qquad (2)$$

$$EP(t) = NA(t)[1 - \prod_{i \in I_2} (1 - \frac{EB_i(t)}{NA(t)})] \qquad (3)$$

and

$$\begin{aligned}ENP(t) &= NA(t)\{1 - \prod_{i \in I_1} (1 - \frac{EB_i(t)}{NA(t)})\}\{1 - \prod_{i \in I_2} (1 - \frac{EB_i(t)}{NA(t)})\} \\ &= \frac{EN(t)\ EP(t)}{NA(t)} . \qquad (4)\end{aligned}$$

To summarize the results, of the NA(t) aircraft, EN(t) cannot fly, [EP(t) - ENP(t)] can fly a limited mission, and the rest, [NA(t) - EN(t) - EP(t) + ENP(t)], can fly any mission. The following Venn diagram also summarizes the results.

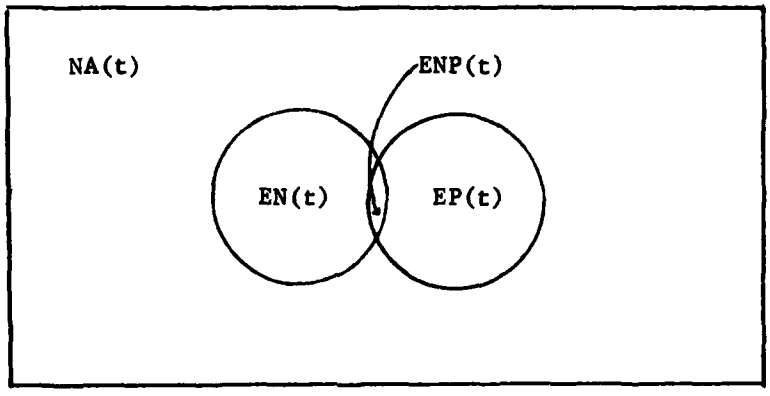

Figure 1.
Venn Diagram for Two Sets of Parts and No Cannibalization

Full Cannibalization

Expressions for EN(t), EP(t), and ENP(t) are again of interest. However, to distinguish the results from those for no cannibalization a subscript of c will be used so expressions will be found for $EN_c(t)$, $EP_c(t)$, and $ENP_c(t)$. The Rand Note [1] gives an expression for computing $P^i(j)$, the cumulative probability that shortages of the i^{th} item are less than or equal to j. The results given here will be in terms of $P^i(j)$.

Since failed components are consolidated onto the smallest number of aircraft, it is important to know how many of the N parts are the same. Let M be the number of different kinds of parts among the N repairable parts making up an aircraft. Also let Q_i be the number of parts of type i. Thus $\sum_{i=1}^{M} Q_i = N$. Since failed components are consolidated onto the smallest number of aircraft, the probability that the number of aircraft NMCS is less than or equal to j is given by

$$P_N(j) = \prod_{i \in I_1} P^i(Q_i \cdot j). \tag{5}$$

The average number of aircraft NMCS under full cannibalization is then

$$EN_c(t) = \sum_{j=0}^{NA(t)-1} [1 - P_N(j)]. \tag{6}$$

Similary, if $P_p(j)$ is the probability that the number of aircraft PMCS is less than or equal to j, then

$$P_p(j) = \prod_{i \in I_2} P^i(Q_i \cdot j). \tag{7}$$

The average number of PMCS aircraft under full cannibalization is then

$$EP_c(t) = \sum_{j=0}^{NA(t)-1} [1 - P_p(j)]. \tag{8}$$

Because of the full cannibalization assumption, NMCS aircraft will become a source of parts from the PMCS class, I_2, as needed. Thus an NMCS

aircraft can also become PMCS since it can provide parts from set I_2. Similarly a PMCS aircraft can become NMCS by serving as a source of parts from set I_1. Therefore, the average number of aircraft that are both NMCS and PMCS is

$$ENP_c(t) = \min\{EN_c(t),\ EP_c(t)\}. \tag{9}$$

In summary, under the full cannibalization policy, $EN_c(t)$ aircraft cannot fly. Of the remainder, $\max\{EP_c(t) - EN_c(t),\ 0\}$ can fly a limited mission and $[NA(t) - \max\{EN_c(t),\ EP_c(t)\}]$ can fly any mission. The condition of the aircraft at time t can be characterized by the following Venn diagram, where, for illustration, $EP_c(t) > EN_c(t)$.

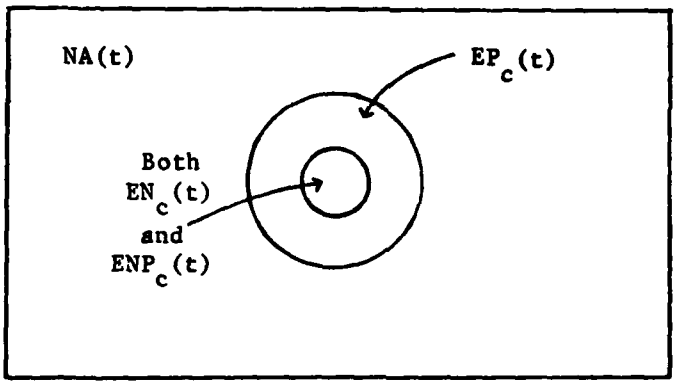

Figure 2.
Venn Diagram for Two Sets of Parts and Full Cannibalization

Comparison of Results for the Two Maintenance Policies

The following numerical example illustrating the results for two sets was suggested by Lt Col Mitchell of the Logistics Management Center. Although the results presented in the paper involve time averages, results for the example are computed for one instant in time. As one might expect, or hope, the relationships among the quantities EN(t), EP(t), and ENP(t) hold for an instant as well.

Suppose that the inventory and repair system consists of five airplanes each composed of five different parts with no spares available. Two of the parts belong to set I_1 (are essential) and three belong to set I_2 (are nonessential). First consider the no cannibalization policy. The status of the parts for all aircraft is given in Table 1 where an "ok" means the part is good and an "X" means the part has failed. Table 1 also gives the resulting mission status of each aircraft as NMCS, PMCS or Fully Mission Capable (FMC).

		Plane				
Set	Part	1	2	3	4	5
I_1	1	ok	ok	X	ok	ok
	2	X	ok	ok	ok	ok
I_2	3	ok	ok	ok	ok	ok
	4	ok	X	X	ok	ok
	5	ok	ok	ok	X	ok
Mission Status		NMCS	PMCS	NMCS and PMCS	PMCS	FMC

Table 1.

Parts and Aircraft Status under No Cannibalization

Figure 3 summarizes the status of the five aircraft based on the information in Table 1. For each set, an aircraft is categorized as up when no parts from that set have failed or down when at least one part from that set has failed. The upper left box gives the number of fully mission capable aircraft since all parts are working. The lower right-hand box gives the number of aircraft both NMCS and PMCS since at least one part from each set has failed. The

marginal entry for I_1 - down gives the number of aircraft NMCS and the marginal entry for I_2 - down gives the number of aircraft PMCS.

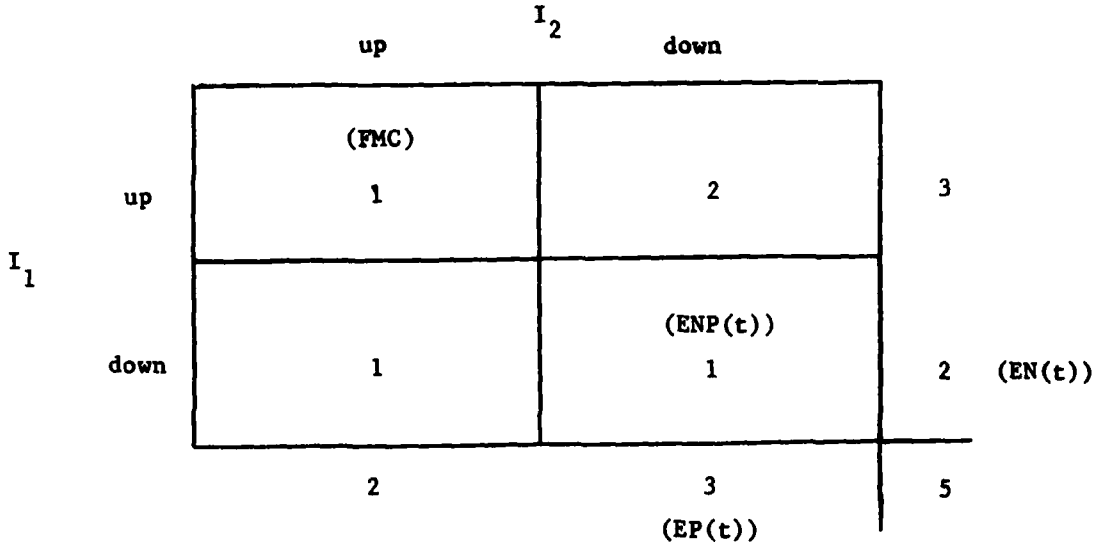

Figure 3.

Aircraft Status under No Cannibalization

Under the full cannibalization policy, the failed parts from both sets are consolidated onto the fewest aircraft. Table 2 gives the status of the parts and aircraft under full cannibalization.

		Plane				
Set	Part	1	2	3	4	5
I_1	1	X	ok	ok	ok	ok
	2	X	ok	ok	ok	ok
I_2	3	ok	ok	ok	ok	ok
	4	X	X	ok	ok	ok
	5	X	ok	ok	ok	ok
Mission Status		NMCS and PMCS	PMCS	FMC	FMC	FMC

Table 2.

Parts and Aircraft Status under Full Cannibalization

Figure 4 summarizes the status of the five aircraft based on the information in Table 2. Note that the full cannibalization

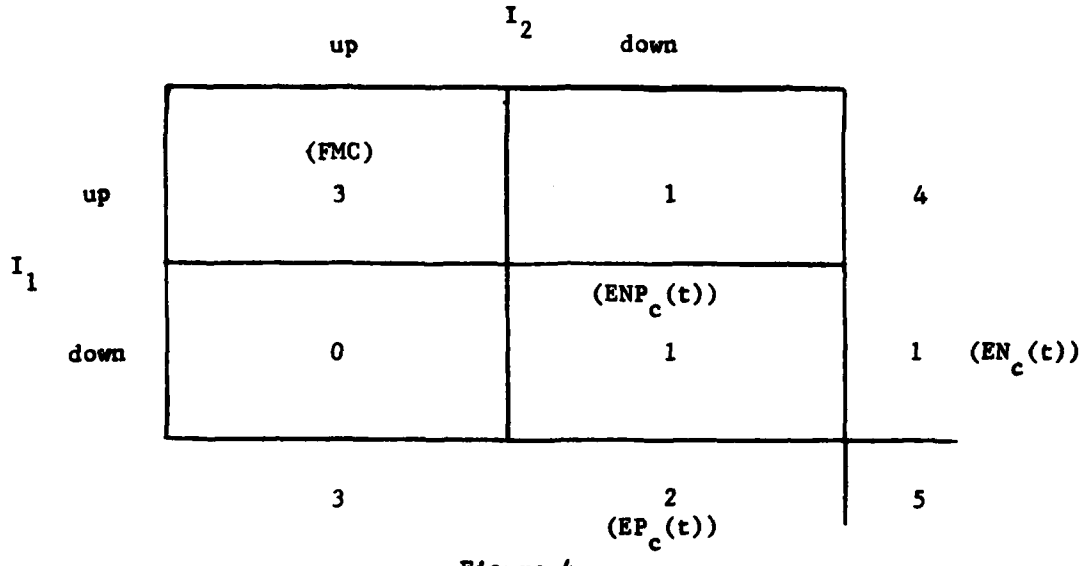

Figure 4.

Aircraft Status under Full Cannibalization

policy leads to an improvement in the status of the aircraft. Of course, there is a price for this improvement--the extra maintenance man-hours required to move parts from one aircraft to another.

SECTION 3

SEVERAL SETS OF PARTS

The assumption that an aircraft can still be flown on a limited mission if several parts from set I_2 have failed is often unrealistic. For example, an aircraft might be able to fly with one of two radios down but not with both down. Thus set I_2, the set of nonessential items, must be subdivided into smaller sets. The way in which I_2 is subdivided depends on the redundancy of aircraft systems and policies concerning flying aircraft with failed parts. For simplicity, assume that an aircraft has R redundant systems each composed of a primary system and a backup, where the failure of any of the several parts that make up the primary (backup) system causes it to fail. Also assume that the flying policy allows an aircraft to be flown if only one of the primary and backup systems is operating for each of the R redundant systems. If both the primary and backup systems fail for any of the R redundant pairs, the aircraft is NMCS. Thus I_2 would be divided such that one set would be truly nonessential items. Also, each primary and each backup system for each redundant pair would have its own set (2R in all). These sets would be paired so that the failure of at least one part in each set of a pair would cause an aircraft to be NMCS.

Computations become difficult if an aircraft has many pairs of redundant systems so only a simple example will be studied. For example, assume the aircraft parts can be assigned to four mutually exclusive sets. I_1 is the set of essential aircraft parts, I_2 is the set of truly nonessential items, I_3 contains the parts that would cause the primary system of the redundant pair to fail, and I_4 contains the parts that would cause the backup system

to fail. Thus, sets I_3 and I_4 are paired so that a failure of one or more parts in each set will cause an aircraft to be NMCS (PMCS, too).

No Cannibalization

The probability that an aircraft is NMCS due to the failure of parts in set I_1 is

$$\text{Prob}(\text{NMCS}_1(t)) = 1 - \prod_{i \in I_1} \left(1 - \frac{EB_i(t)}{NA(t)}\right)$$

The probability that an aircraft is PMCS due to the failure of parts in set I_j, where $j = 2, 3, 4$ is

$$\text{Prob}(\text{PMCS}_j(t)) = 1 - \prod_{i \in I_j} \left(1 - \frac{EB_i(t)}{NA(t)}\right).$$

For brevity, define the events: $N_1 = \text{NMCS}_1(t)$ and $P_j = \text{PMCS}_j(t)$.

Since an aircraft is NMCS if a failure occurs in parts from set I_1 or from both I_3 and I_4, the overall probability that an aircraft is NMCS at time t is

$$\begin{aligned}\text{Prob}(\text{NMCS}(t)) &= \text{Prob}\{N_1 \cup (P_3 \cap P_4)\} \\ &= \text{Prob}(N_1) + \text{Prob}(P_3 \cap P_4) - \text{Prob}(N_1 \cap P_3 \cap P_4) \end{aligned} \quad (10)$$

Note that the independence of failures allows Equation (10) to be written as

$$\begin{aligned}\text{Prob}(\text{NMCS}(t)) &= \text{Prob}(N_1) + \text{Prob}(P_3) \cdot \text{Prob}(P_4) \\ &\quad - \text{Prob}(N_1) \cdot \text{Prob}(P_3) \cdot \text{Prob}(P_4) \end{aligned} \quad (11)$$

The probability that an aircraft is PMCS at time t is

$$\begin{aligned}\text{Prob}(\text{PMCS}(t)) &= \text{Prob}(P_2 \cup P_3 \cup P_4) = \text{Prob}(P_2) + \text{Prob}(P_3) + \text{Prob}(P_4) \\ &\quad - \text{Prob}(P_2 \cap P_3) - \text{Prob}(P_2 \cap P_4) - \text{Prob}(P_3 \cap P_4) \\ &\quad + \text{Prob}(P_2 \cap P_3 \cap P_4) \end{aligned} \quad (12)$$

Since failures are independent, Equation (12) can be written in terms involving only $\text{Prob}(P_2)$, $\text{Prob}(P_3)$ and $\text{Prob}(P_4)$.

The probability that an aircraft is FMC at time t is

$$\text{Prob}(\text{FMC}(t)) = (1 - \text{Prob}(N_1)) \prod_{j=2}^{4} (1 - \text{Prob}(P_j)) \tag{13}$$

The probability that an aircraft is both NMCS and PMCS at time t is given by

$$\text{Prob}(\text{NMCS}(t) \cap \text{PMCS}(t)) = \text{Prob}(\text{FMC}(t)) + \text{Prob}(\text{NMCS}(t)) + \text{Prob}(\text{PMCS}(t)) - 1 \tag{14}$$

These probabilities together with the number of aircraft supported at time t, NA(t), can be used to compute the desired averages EN(t), EP(t), and ENP(t). Thus

$$\begin{aligned} \text{EN}(t) = \text{NA}(t)[&\text{Prob}(N_1) + \text{Prob}(P_3) \cdot \text{Prob}(P_4) \\ &- \text{Prob}(N_1) \cdot \text{Prob}(P_3) \cdot \text{Prob}(P_4)], \end{aligned} \tag{15}$$

$$\begin{aligned} \text{EP}(t) = \text{NA}(t)[&\text{Prob}(P_2) + \text{Prob}(P_3) + \text{Prob}(P_4) - \text{Prob}(P_2) \cdot \text{Prob}(P_3) \\ &- \text{Prob}(P_2) \cdot \text{Prob}(P_4) - \text{Prob}(P_3) \cdot \text{Prob}(P_4) \\ &+ \text{Prob}(P_2) \cdot \text{Prob}(P_3) \cdot \text{Prob}(P_4)] \end{aligned} \tag{16}$$

and

$$\begin{aligned} \text{ENP}(t) &= \text{NA}(t)[\text{Prob}(\text{NMCS}(t)) \cap \text{Prob}(\text{PMCS}(t))] \\ &= \text{NA}(t)\text{Prob}(\text{FMC}(t)) + \text{EN}(t) + \text{EP}(t) - \text{NA}(t) \end{aligned} \tag{17}$$

In summary, of the NA(t) aircraft, EN(t) cannot fly, [EP(t) - ENP(t)] can fly a limited mission, and the rest, [NA(t) - EN(t) - EP(t) + ENP(t)] can fly any mission.

Full Cannibalization

Let $P_k(j)$ be the probability that the number of aircraft short of parts in set I_k is less than or equal to j. Thus

$$P_k(j) = \prod_{i \in I_k} P^i(Q_i \cdot j)$$

Denoting the average number of aircraft short of parts from set I_k as $EI_k(t)$ and following the Rand Note yields

$$EI_k(t) = \sum_{j=0}^{NA(t)-1} [1 - P_k(j)].$$

Again, let $EN_c(t)$, $EP_c(t)$, and $ENP_c(t)$ be the average number of aircraft NMCS, PMCS, and both NMCS and PMCS, respectively, when full cannibalization is allowed. Aircraft are NMCS if parts in set I_1 have failed or if parts in both sets I_3 and I_4 have failed. Thus

$$EN_c(t) = \max \{E_1(t), \min [EI_3(t), EI_4(t)]\} \quad (18)$$

Since failed parts are consolidated onto the smallest number of aircraft, the failure of parts in both sets I_3 and I_4 can contribute to the number of NMCS aircraft only if the number of aircraft with failed parts from set I_3 and from set I_4 is greater than the number of aircraft with failed parts from set I_1.

The expected number of aircraft PMCS at time t can be found from

$$EP_c(t) = \max\{EI_2(t), EI_3(t), EI_4(t)\}. \quad (19)$$

The average number of aircraft that are both NMCS and PMCS is

$$ENP_c(t) = \min\{EN_c(t), EP_c(t)\} \quad (20)$$

In summary, of the NA(t) aircraft, $EN_c(t)$ cannot fly $[EP_c(t) - ENP_c(t)]$ can fly a limited mission, and the rest, $[NA_c(t) - EN_c(t) - EP_c(t) + ENP_c(t)]$ can fly any mission.

Comparison of Results for the Two Maintenance Policies

As in Section 2, the results will be compared at an instant in time for a numerical example. Suppose that the inventory and repair system consists of six airplanes each composed of four different parts with no spares available. One part is essential and belongs to set I_1; another is nonessential and belongs to I_2. The last two parts form a redundant pair and thus are assigned one to set I_3 and the other to I_4.

First consider the no cannibalization policy. The status of the parts for all aircraft is given in Table 3 where an "ok" means the part is good and an "X" means the part has failed. Table 3 also gives the resulting mission status of each aircraft as NMCS, PMCS, or FMC.

		Plane					
Set	Part	1	2	3	4	5	6
I_1	1	X	ok	ok	ok	ok	X
I_2	2	ok	X	ok	ok	X	ok
I_3	3	ok	ok	X	X	ok	X
I_4	4	ok	ok	X	ok	X	ok
Mission Status		NMCS	PMCS	NMCS and PMCS	PMCS	PMCS	NMCS and PMCS

Table 3.

Parts and Aircraft Status under No Cannibalization
(Several Sets of Parts)

Under the full cannibalization policy, the failed parts from both sets are consolidated onto the fewest aircraft. Table 4 gives the status of the parts and aircraft under full cannibalization.

		Plane					
Set	Part	1	2	3	4	5	6
I_1	1	X	X	ok	ok	ok	ok
I_2	2	X	X	ok	ok	ok	ok
I_3	3	X	X	X	ok	ok	ok
I_4	4	X	X	ok	ok	ok	ok
Mission Status		NMCS and PMCS	NMCS and PMCS	PMCS	FMC	FMC	FMC

Table 4.

Parts and Aircraft Status under Full Cannibalization
(Several Sets of Parts)

Table 5 compares the results of the two maintenance policies. The entries give the number of planes with a given status under each policy. As in Section 2, the full cannibalization policy gives better aircraft status than no cannibalization does.

	Maintenance Policy	
Mission Status	No Cannibalization	Full Cannibalization
NMCS	3	2
PMCS	5	3
NMCS and PMCS	2	2
FMC	0	3

Table 5.

Comparison of Results for No Cannibalization and Full Cannibalization
(Several Sets of Parts)

SECTION 4

CONCLUSION

The assumption in Dyna-METRIC that the failure of any aircraft part would cause the aircraft to be NMCS is unrealistic. Some relaxation of that assumption is achieved in this paper first by categorizing parts as essential and nonessential and later by further dividing the nonessential category. In the simple case of only two classes of parts the results are quite tractable under both no cannibalization and full cannibalization assumptions. Expressions are found for the expected number of aircraft NMCS, PMCS, and both NMCS and PMCS. These expressions allow computation of the number of aircraft that can fly any mission, limited missions, and no mission.

A numerical example suggested by Lt Col Mitchell of the LMC illustrates the relationship between the numbers of aircraft NMCS, PMCS, or both and the numbers of aircraft that can fly no mission, a limited mission, or any mission. The numerical example also points out the advantage of full cannibalization over no cannibalization. This advantage is due to the ability to move failed parts to other aircraft but would, in practice, demand lots of manpower to carry out.

Making the model more realistic by further dividing the class of parts which cause an aircraft to be PMCS leads to more difficult expressions for the numbers of aircraft NMCS, PMCS, and both. Since the complexity of the expressions grows with the number of classes of parts, only a simple example was developed. With several classes of PMCS parts, expressions could be developed for the number of aircraft PMCS due to parts from each class. Thus various categories of limited missions could be developed for the aircraft.

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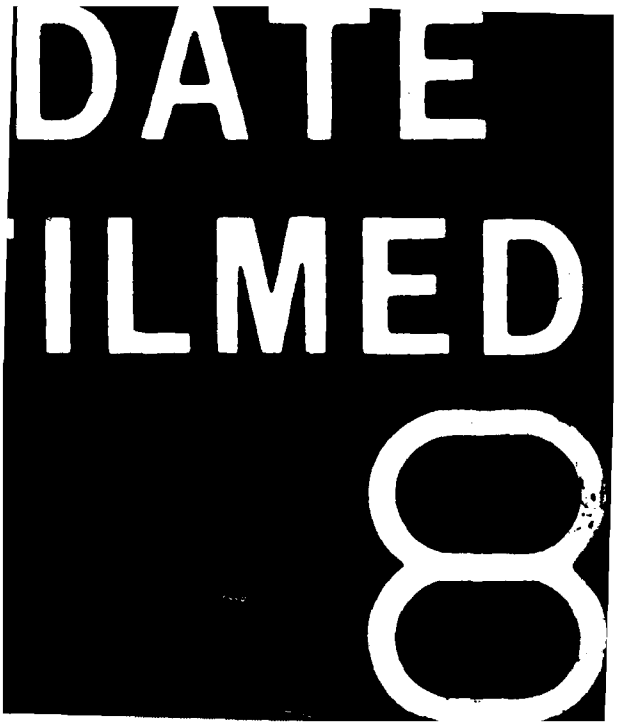
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